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VERIFICATION OF DYNAMIC LOAD FACTOR FOR ANALYSIS OF AIRBLAST-LOADED MEMBRANE SHELTER PANELS BY NONLINEAR FINITE ELEMENT CALCULATIONS

By
Thomas A. Godfrey

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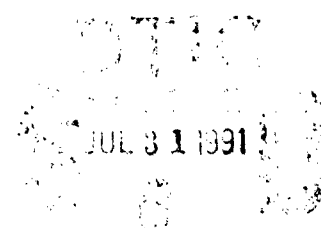
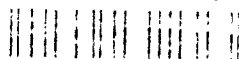
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13. ABSTRACT (Maximum 200 words) A structural concept consisting of thin membrane skin panels supported by stiffening ribs is under study for use in tactical shelters designed to withstand airblast. For design analysis it is desirable to evaluate membranes under static loadings in place of more complex and costly dynamic analysis. Previous analysis indicated that membrane panel deflections produced by the dynamic airblast loading would be approximately equal to those produced by a static pressure loading four times as great. This result is confirmed in work reported here using a nonlinear finite element code to determine the static and dynamic response of rectangular and infinitely long membranes. The use of static loads of four times the peak airblast-reflected overpressure loading should be sufficient for design purposes in consideration of head-on blast encounters with the membrane panel.				
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Preface

This work was performed from February to September 1990 as part of a work unit "Advanced Shelter Hardening Concepts" of the Tactical Shelter 6.2 Program. The work was undertaken to confirm a result¹ obtained by John C. Brewer concerning the dynamic response of membrane panel structures to instantaneously applied pressure loads. A prototype shelter was designed with membrane panels using a dynamic load factor of two in the sizing of the membrane panels. Brewer found that the factor of two is incorrect and that the correct dynamic load factor is actually four. The author wishes to acknowledge John Brewer's contributions to this effort. This effort was funded under Program Element 62786, Project 1L162786A427, Task AA, Work Unit Accession A00.

VERIFICATION OF DYNAMIC LOAD FACTOR FOR ANALYSIS OF AIRBLAST-LOADED MEMBRANE SHELTER PANELS BY NONLINEAR FINITE ELEMENT CALCULATIONS

Introduction

In developing lightweight tactical shelters capable of withstanding the effects of airblast, a number of alternative structural concepts were considered. A wall construction concept consisting of thin membrane panels supported by stiffening frames is potentially very lightweight. For design analysis it is desirable to evaluate membranes under static loadings in place of more complex and costly dynamic analysis. The nonlinear membrane response, however, suggests that the use of standard dynamic load factors derived for linear systems is improper.

Prior work¹ considering the dynamic response of rectangular membranes found that, for membranes of interest in the design of shelters, peak deflections and stresses produced by the dynamic airblast loading would be approximately equal to those produced by a static pressure loading four times as large. In this report, static and dynamic finite element calculations are presented which theoretically confirm this result.

Two membrane geometries are considered: rectangular and infinitely long. The infinitely long geometry is a membrane of finite width, but infinite length. The membranes are similar to the E-glass/epoxy membrane panels in a prototype blast-hardened composite shelter that was fabricated and evaluated in a high explosive blast test. The membranes studied here, however, have quasi-isotropic material properties. A readily available finite element code, NONSAP-C, is used to determine the dynamic response of the membranes by direct integration of the equations of motion. The dynamic results are compared to results of static analysis to confirm the dynamic load factor. Results obtained from NONSAP-C are compared to results from NISA-II, a large scale commercial finite element program, and an analytic solution for the infinitely long membrane.

Methods

A version of NONSAP modified for analysis of concrete structures, NONSAP-C,² was readily available and had the capabilities required for this effort. NONSAP-C was obtained and modified to run on a Sun-4 workstation.

NONSAP-C performs nonlinear static and dynamic analysis through step-by-step solution of the system of equations, updating the system matrices and performing equilibrium iterations to achieve convergence at each load or time step. Dynamic problems are solved by direct integration of the equations of motion using either the Wilson-Theta or the Newmark-Beta integration algorithms². Calculations performed in this work used the Wilson-Theta algorithm.

A shortcoming of NONSAP-C is that the load vector is not updated during the incremental solution of the system of equations to account for changes in geometry. Pressure loads applied to element surfaces in the model therefore always act in the direction normal to the undeformed geometry of the model. Since the membranes will undergo relatively large rotations under the airblast loading, this departs from a completely realistic analysis. For the response anticipated, the component of the pressure load normal to the original geometry will be considerably larger than the component tangent to the original geometry over most of the membrane's surface. Therefore, the lateral loading of the membrane, in lieu of a true pressure loading, is a useful approximation for exploring the relationship of the static and dynamic response. For consistency in this work, all pressure loads are treated as loads normal to the undeformed geometry of the membrane.

The NONSAP-C code offers a limited selection of general purpose structural and continuum elements. NONSAP-C's three dimensional continuum element is the only element provided capable of modeling the nonlinear membrane behavior of thin skins. The three dimensional continuum element is an isoparametric eight- to 21-node element with three displacement degrees of freedom per node. For the calculations performed here, eight-node, 12-node and 16-node elements were used. The eight-node element interpolates element geometry and displacements linearly. Additional nodes are added to the mid-point of an edge to allow parabolic interpolation along the edge. The calculations performed with 12- and 16-node elements (four and eight mid-side nodes) resulted in greater accuracy with significantly fewer degrees of freedom.

Two membrane geometries were studied: infinitely long and rectangular. The infinitely long membrane has finite width, but infinite length. This geometry is useful for the consideration of long, narrow membrane panels. The width of the infinitely long membrane approximates the width of membrane of panels employed in a prototype shelter. The rectangular membrane studied is 6.5 in wide by 38 in long, the approximate dimensions of the prototype membrane panels.

The membrane skin model has quasi-isotropic material properties and a nominal thickness of 0.0275 in. The material properties and density are typical of E-glass/epoxy materials, but no particular material or lay-up is modeled. For the model, $E=3.01$ msi, $\nu=0.25$, and $\rho=1.7E-4$ s² lb/in⁴.

Quarter symmetry is used in the rectangular membrane model and half symmetry is used in the infinitely long membrane model. The infinitely long membrane is modeled by a strip of elements running across the width of the membrane. Nodal displacements are permitted in the widthwise and transverse directions, but no displacements are permitted in the lengthwise direction as a consequence of the infinite length. Three-dimensional continuum elements are used in a single layer the same thickness as the membrane. The top and bottom nodes of the elements are fixed at the membrane edge

boundary; thus, the membrane edges are constrained from rotating.

The clamped edge condition imposes a modeling difficulty. Since the membrane has negligible bending stiffness, a clamped membrane will undergo large rotations within a short distance from the clamped edge. The three-dimensional elements have to be severely distorted to accommodate the large rotations. This suggests that either a small mesh size, or higher order polynomial interpolation, or both, should be used in the edge region. Calculations performed with eight-node elements made use of a mesh with two refinements in the mesh size near the edge. Nearest the edge are five elements that together span a distance equal to the membrane thickness. The next five larger elements build the model out to a distance of 1/10 the model's width or length dimension. The remaining model is composed of elements 1/10 of the model's width/length dimension in size. This geometry, which will be referred to as the refined mesh in the results section, was used for both rectangular membranes and infinitely long membranes. Infinitely long membranes were also modelled using a mesh of 120 eight-node elements. The 120-element mesh provides elements that are approximately the same dimension in length, width, and thickness.

The higher order 12- and 16-node elements were used for the infinitely long and rectangular membrane calculations, respectively. Calculations performed with higher order elements used meshes with uniform element size.

An analytical solution is developed for the nonlinear deflection of the infinitely long membrane based on the equilibrium equation for a stretched, laterally loaded flexible string.³ Numerical results are obtained by solution of a nonlinear algebraic equation. Development of the solution is given in the Appendix.

The NISA-II finite element code is used as an independent check of results obtained with NONSAP-C. The general thin shell element is used to model the membrane. This element has six degrees of freedom per node, allowing rotation of the membrane edges.

The pressure loading considered is that originating from a 10 psi free field overpressure airblast encountering the membrane panel. The direction of travel of the shock front is normal to the surface of the membrane. In such an encounter, the pressure rises rapidly to 25.3 psi, arising from reflection of the shock wave off the membrane surface. Test data⁴ indicate a rise time of 0.18 ms for a shock wave encounter similar to that being considered. The average pressure on the panel remains close to 25.3 psi for a millisecond, then decays to approximately 12 psi over the next 10 to 20 milliseconds.

Calculations were performed using a dynamic load model consisting of an instantaneously applied pressure of 25.3 psi, held constant for the time period of the calculation. Assuming an instantaneous load application is the usual practice in analysis of a blast response of shelter wall structures. In this case, however, the

rise time of the pressure load will be a significant fraction of the time-to-peak response of the membrane. Since membrane peak response is expected to occur at about a half millisecond after blast arrival, the pressure would not reach its peak until almost 40% of the time to peak response of the membrane has expired, assuming the 0.18 ms rise time⁴ applies. The use of the instantaneous rise time seems overly conservative, but rough calculations indicate that the differences in peak response between the instantaneously applied load and the load with the finite rise time are less than 1%. This result is because during the first 40% of the time-to-peak response the membrane is being accelerated from rest and deflects only a small amount. When the pressure has risen to its full 25.3 psi, it is acting on a membrane panel that has a deflection and velocity small enough to result in a peak response similar to that caused by an instantaneously applied pressure.

The assumption made here of instantaneous load application borders on being inappropriate for the specific structures studied in this effort. Membranes under consideration in shelter design are generally wider than the narrow membranes studied here. Wider, more massive membranes will take longer to reach peak response. Hence, the results obtained considering an instantaneously applied load should apply for the larger membranes under consideration.

Results

Infinitely long membranes under static loading.

For the infinitely long, 6.5 in wide membrane, deflections and stresses were determined for a static load of 101.2 psi, based on a dynamic load factor of four. Calculations were performed using a variety of NONSAP-C models, NISA-II, and an analytical solution. Mid-point deflections are in Table 1.

Table 1 Static Deflections of Infinite Membranes

Model	Mid-Point Deflection (in)
19 8-node elements, refined mesh	0.446
120 8-node elements	0.451
20 12-node elements	0.452
NISA-II 99 general shell elements, edge clamped	0.452
NISA-II 99 general shell elements, edge simply supported	0.457
Analytical solution	0.464

The following nonlinear equation was solved iteratively to numerically obtain the mid-point deflection for the analytical solution. The derivation is given in the Appendix.

$$\frac{2\delta A_{11}(L_0^1 - 1)}{1^3 + \frac{4}{3}\delta^2 1} = q \quad L_0^1 = \int_0^1 \sqrt{1 + \left(\frac{2\delta x}{1^2}\right)^2} dx$$

Here δ is mid-point deflection, 1 is one half the membrane width, q is the pressure load, and A_{11} is the membrane's extensional stiffness.

The deflections obtained using 120 eight-node elements, 20 12-node elements, and NISA-II (99 elements, edge clamped) agree well. The NONSAP-C 120 eight-node element model took a much longer time to run than the models using higher order elements, however, indicating the greater efficiency of using higher order elements. The NISA-II results were expected to be the most accurate, because the general shell elements used, which interpolate rotations in addition to displacements, are effectively higher order elements than the NONSAP-C continuum elements. Since the NONSAP-C result and the NISA-II result for clamped edge conditions are identical to three places despite the large difference in number and effective

order of the elements used, the use of parabolic interpolation with a mesh size of 1/20 of the membrane's width is considered efficient and accurate.

The difference between the NISA-II results for clamped edge conditions and simply supported edge conditions is about 1%. This indicates that for this loading level, the response is predominately membrane response, with little load being carried in bending. The analytical solution differs from the NISA-II simply supported result by only 1.5%. This can be attributed to the small amount of bending resistance provided by the laminate.

The widthwise (x) direction is the most highly stressed in the infinitely long membrane. The NONSAP-C code calculates a three-dimensional stress state. In this three-dimensional calculated stress state, σ_y is roughly $v\sigma_x$, due to the y direction boundary conditions, and σ_z is relatively small. Hence, only the widthwise stresses (σ_x) calculated are given in Table 2. This three dimensional state of stress is somewhat inaccurate because the quasi-isotropic properties being used do not model the actual composite laminate's elastic properties in the z direction.

Table 2 Static Stresses in Infinitely Long Membranes.

Model	σ_x at edge (ksi)	σ_x max/min point between edge & mid-point (ksi)	σ_x at mid-point (ksi)
20 12-node elements	40.4	39.6	40.9
NISA-II 99 elements, edge clamped	38.9	41.9	40.2
NISA-II 99 elements, simply supported	43.3	-	41.7
Analytical solution	41.9	-	40.3

The membrane stress for the analytical solution was calculated from the following expression for the stress resultant derived in the Appendix.

$$N_x = \frac{A_{11}(L_0^2 - 1)}{1^3 + \frac{4}{3}\delta^2 1} \sqrt{1 + \frac{4\delta^2 x^2}{1^4}}$$

The calculated stresses for the simply supported edge condition

agree well with the analytical solution. The NISA-II simply supported result and the analytical solution show the stress being a maximum at the edge, and a minimum at the mid-point. This behavior is the consequence of equilibrium of the pressure loaded membrane, given that the pressure load acts only in the original, out-of-plane direction and does not follow the deformed surface. The results for the clamped edge condition modelled by NONSAP-C and by NISA-II are more difficult to interpret, but the general magnitude of the stresses calculated appears reasonable.

Dynamic response of infinitely long membranes.

The dynamic response of infinitely long membranes to an instantaneously applied pressure of 25.3 psi was determined through dynamic analysis using NONSAP-C. A time step of 0.01 ms was used in the time integration. Figure 1 gives the membrane deflection surface at $t=0.30$, 0.40, and 0.45 ms, and the static deflection surface under the 101.2 psi load. The x axis in Figure 1 goes from 0 at the edge to 3.25 in at the membrane mid-point. The results are for the 120 eight-node element model.

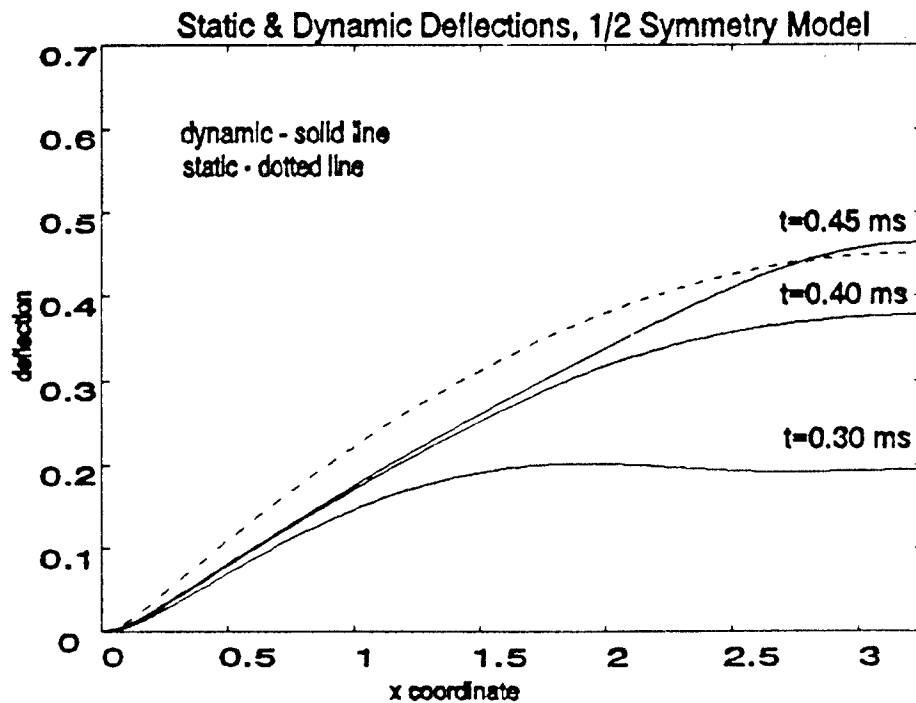


Figure 1 Static and Dynamic Deflections of Infinite Membrane, 120 eight node element model.

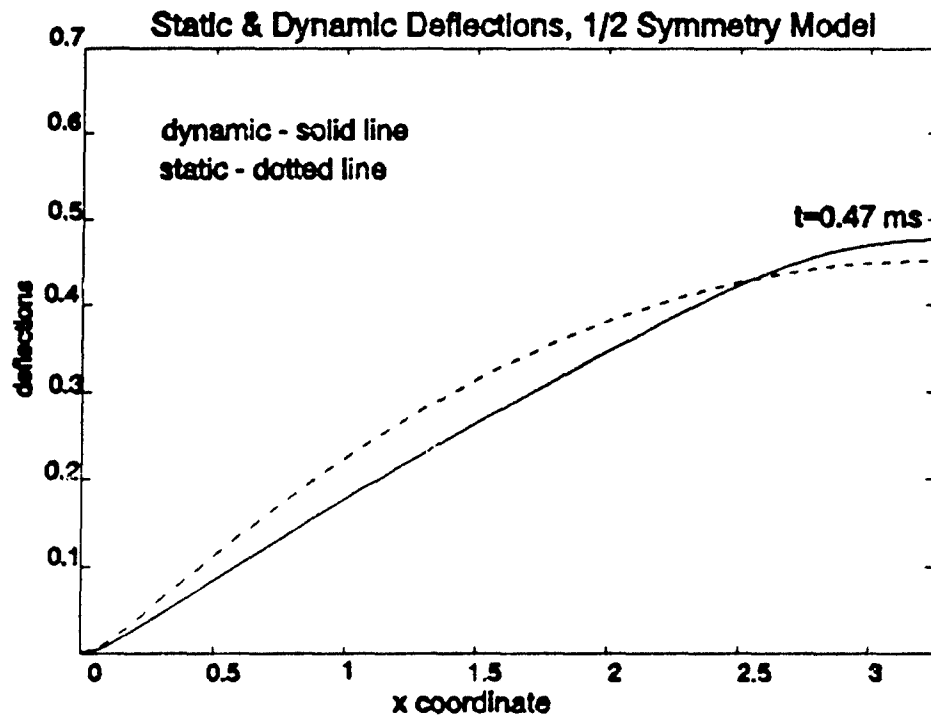


Figure 2 Static and Peak Dynamic Deflections of Infinite Membrane, 20 12-node element model.

Figure 2 gives the membrane peak deflected surface at $t=0.47$ ms, and the static deflected surface under a load of 101.2 psi. Results are for the 20 12-node element model.

Table 3 gives widthwise stresses and midpoint deflections for the peak dynamic response of the membrane compared with some of the previously reported static data from Tables 1 and 2.

Table 3 Dynamic and Static Deflections and Stresses.

Model	mid-point deflection (in)	σ_x at edge (ksi)	local max / min σ_x (ksi)	σ_x at mid-point (ksi)
dynamic, 25.3 psi, 20 12-node elements, t=0.47 msec	0.477	38.0	38.3 / 37.8	38.3
static, 101.2 psi, 20 12-node elements	0.452	40.4	39.6	40.9

From Figure 2, there are clear differences in the peak dynamic deflected shape and the deflected shape produced by the static load incorporating the dynamic load factor of four. This difference may be less pronounced if pressure loads normal to the deformed geometry are applied instead of loads normal to the original geometry. In a strict sense, the trial static equivalent load does not reproduce the peak dynamic response of the membrane. For engineering design purposes, however, the static equivalent load reproduces the pertinent response features to within acceptable levels. From Table 3, mid-point deflections differ by 5% and stresses differ by less than 7%. For most applications, material properties may vary by $\pm 7\%$. The static equivalent load provides an acceptable calculation of the deflection and stresses in the dynamically loaded membrane.

Rectangular membranes under static loading.

The deflections and stresses in a 6.5 in by 28 in rectangular membrane under a static load of 101.2 psi were calculated using two NONSAP-C models and a NISA-II model. Deflections are indicated in Table 4. The results for each model are consistent to two significant figures, indicating a deflection of 0.45 in.

In each of the calculations reported in Table 4, there is a small physical inconsistency in that the peak deflection and the membrane mid-point deflection are slightly different. Peak deflections occur at nodes lying along the longitudinal centerline of the membrane, between the membrane mid-point and the 6.5 in long edge. For the NONSAP 100 16-node element model and the NISA-II model, these differences are practically insignificant. For the NONSAP 361 8-node element model, the difference is large and may be because of the relative coarseness of the mesh in this model outside of the narrow edge region.

Table 4 Static Deflections of Rectangular Membranes.

Model	mid-point deflection (in)	peak deflection (in)
361 8-node elements, refined mesh	0.446	0.454
100 16-node elements	0.446	0.447
NISA-II 200 elements, edges simply supported	0.453	0.454

The deflected shape of the quarter symmetry model is shown in Figures 3, 4, and 5. Figure 4 is a view showing deflections along the length of the membrane, and Figure 5 shows deflections across the width of the membrane.

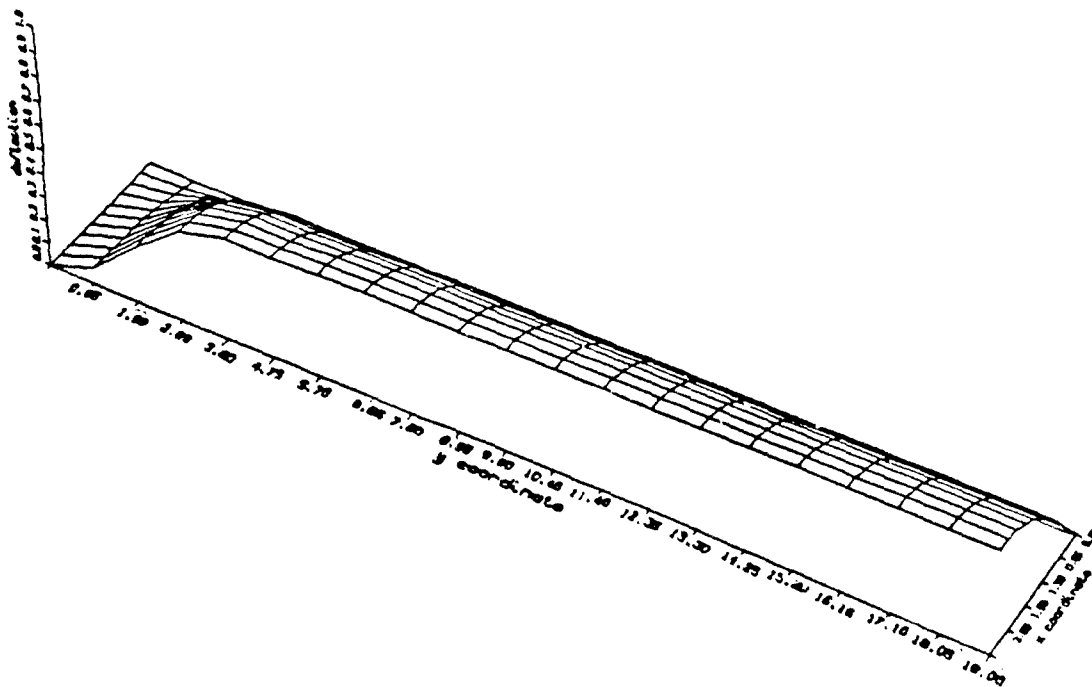


Figure 3 Static Deflection of Rectangular Membrane.

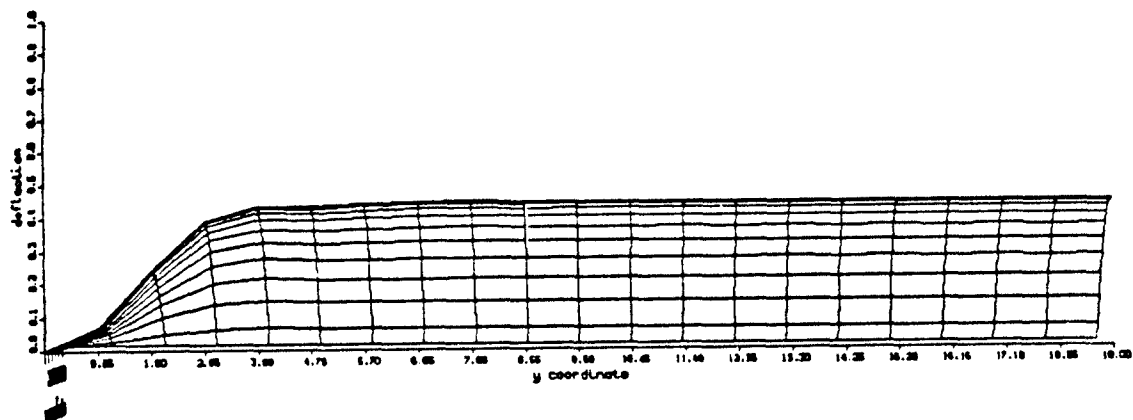


Figure 4. Lengthwise View of Rectangular Membrane Static Deflections.

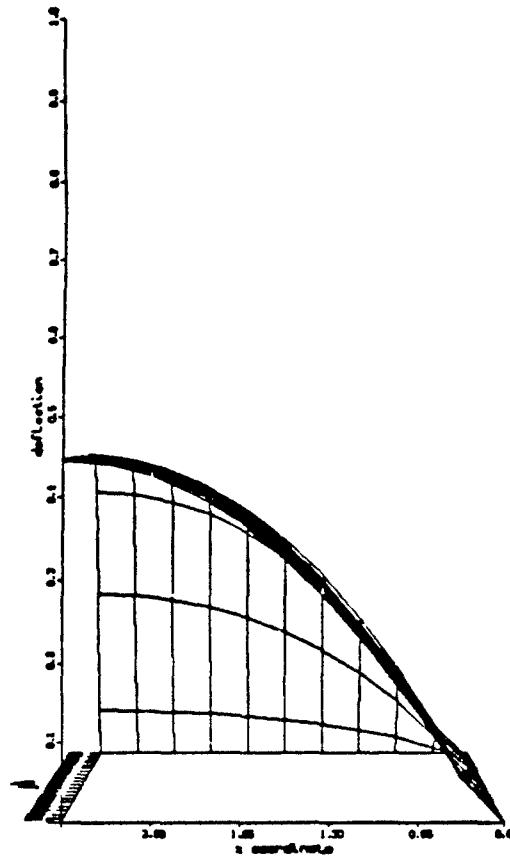


Figure 5 Widthwise View of Rectangular Membrane Static Deflections.

Rectangular membranes under dynamic loadings.

The dynamic response of the rectangular membrane to an instantaneously applied pressure of 25.3 psi was determined through dynamic analysis using NONSAP-C. The 100 element model with 16-node elements used in the static analysis was selected for the dynamic analysis. A time step of 0.01 ms was used in the time integration. A peak dynamic deflection of 0.473 in occurs at 0.47 ms. The peak deflected shape is shown in Figure 6.

As with the dynamic response of the infinitely long membrane, the dynamic deflected shape of the rectangular membrane has a somewhat greater peak deflection than that produced by the static equivalent load. The difference in peak deflection is less than 6%. Hence, the static equivalent load of 101.2 psi reproduces to an acceptable level the peak dynamic response of the membrane to the rapidly applied pressure of 25.3 psi.

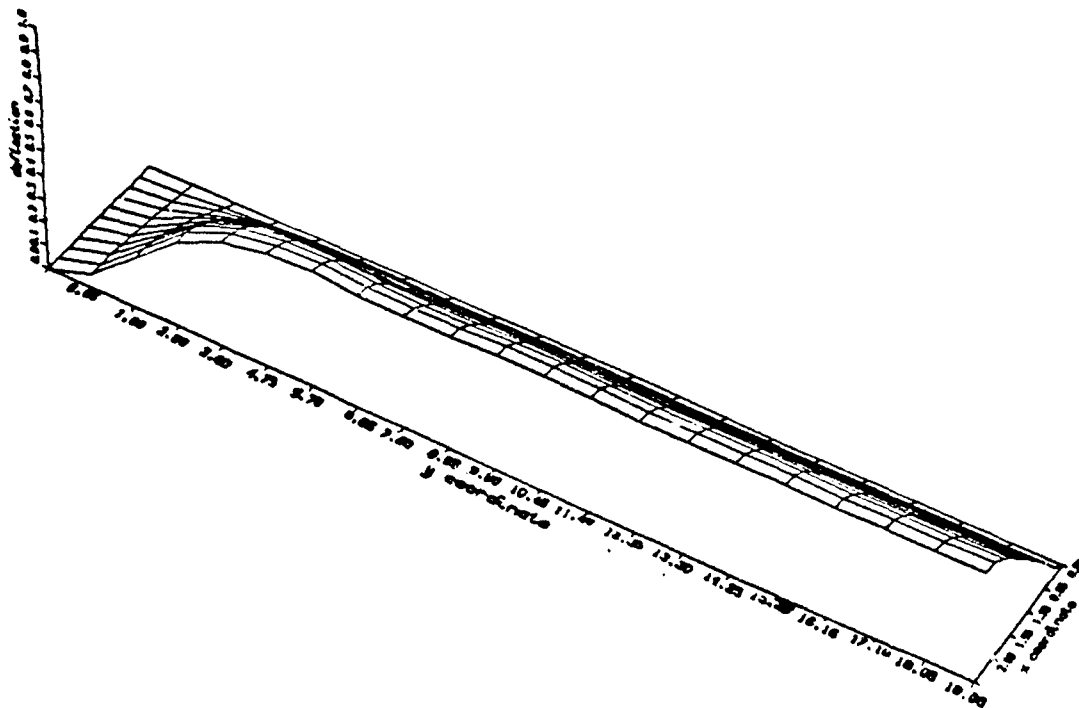


Figure 6 Peak Deflected Shape of Dynamically Loaded Membrane.

Discussion of the analytical solution.

In the Appendix an analytical solution is developed for an infinitely long membrane under a transverse pressure loading. It is shown that the pressure is essentially proportional to the cube of the membrane mid-point deflection. In a preceding section of this note, the analytical solution was used in calculation of the deflection of a particular infinitely long membrane under a static load of 101.2 psi. In Figure 7 the analytical relation between pressure and deflection for the infinitely long membrane is plotted along with the behavior of the cubic term from the Taylor series expansion of the analytical relation.

The difference between the cubic and the analytical solution is small over the range of deflections. At the deflections corresponding to the 101.2 psi load, the difference is about 4%. For smaller deflections the difference is considerably less. Thus the load versus deflection relationship is essentially that of a cubic spring. Brewer¹ shows that for systems with cubic springs the peak dynamic response to an instantaneously applied load is equivalent to the static response to a load four times as great. Therefore, the use of a dynamic load factor of four to reproduce the dynamic response of the membrane to an instantaneously applied pressure load is supported by the analytical model.

Considering the analytical solution to act dynamically as a mass on a cubic spring involves the assumption that the membrane responds dynamically as a single degree of freedom system where the response is fully characterized by the mid-point deflection. The dynamic deflected shape is assumed to always be the static shape associated with a particular mid-point deflection. The dynamic load factor of four derived under this simplification is supported by the finite element calculations in this report. The dynamic finite element calculations permit a high degree of freedom in the transient response of the membrane. Even with the differences permitted in the finite element calculations, the peak dynamic response can still be calculated to within acceptable levels by a static equivalent load using a dynamic load factor of four.

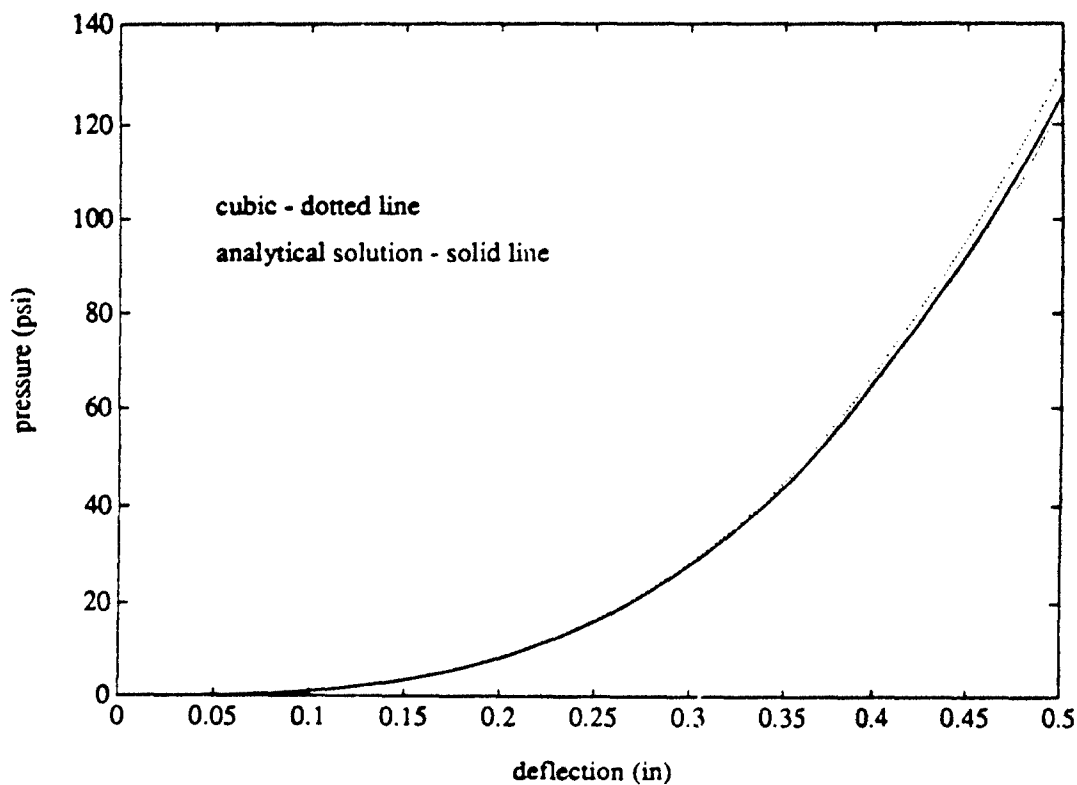


Figure 7 Comparison of Analytical Solution and Cubic Term.

Conclusions

The deflections and stresses for both static and dynamic membrane response to blast loading were numerically calculated. The results indicate that a static load of four times the peak reflected overpressure in the dynamic blast event can be used as a static equivalent load for structural analysis and design. This result applies for head-on blast encounters. This result is valid for the lower order response modes which the 100 element model could capture.

While peak dynamic deflection of the membrane differs in shape from the static equivalent deflection, the differences are small. Mid-point deflections were 5 to 6% greater in the dynamically loaded membrane. Stresses in the dynamically loaded membrane are slightly lower, but within 7% of the stresses in the static equivalent loaded membranes. These differences may be less pronounced if pressure loads normal to the deformed geometry are used in the calculations.

An analytical solution developed for an infinitely long membrane supports a dynamic load factor of four. Taken together, the analytical result and the finite element results provide confidence in the use of the dynamic load factor of four in design analysis of membranes for blast-hardened shelters.

This document reports research undertaken at the US Army Natick Research, Development and Engineering Center and has been assigned No. NATICK/TR 91/041 in the series of reports approved for publication.

References

1. Brewer, John C., Static Analyses of Orthotropic and Non-Linear Pressure Loaded Membranes, paper to be submitted to the International Journal of Solids and Structures.
2. Smith, P. D., Anderson, C. A., "NONSAP-C: A Nonlinear Stress Analysis Program for Concrete Containments Under Static, Dynamic, and Long-Term Loadings", Los Alamos Scientific Laboratory, NUREG/CR-0416, LA-7496-MS, Informal Report, October 1978.
3. Timoshenko, S., Woinowsky-Krieger, S., Theory of Plates and Shells, second edition, copyright 1959, article 83., page 351-352.
4. Tilson, David L., Data Package for Nuclear Hardened Lightweight HMMWV Shelter, Defense Nuclear Agency Experiment No. 1001 MISERS GOLD Event, US Army Engineer Waterways Experiment Station, July 1989.

APPENDIX

Analytical Solution

Consider an infinitely long membrane of finite width with a transverse uniform loading. Such a membrane would deflect to a cylindrical surface. The deflections can be studied by analysis of a widthwise strip of the membrane.

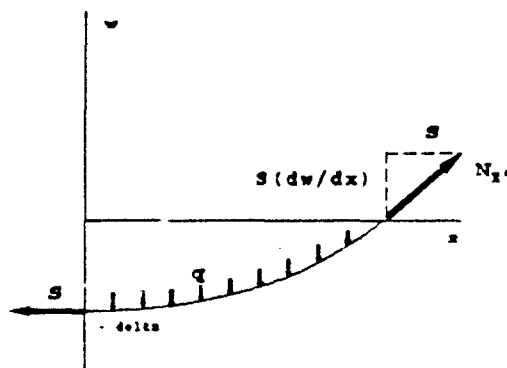
We develop a nonlinear algebraic equation relating the transverse loading, q , to the mid-point deflection, δ , of the strip. This equation can be solved numerically to obtain δ for a specific q . We will also show that the relationship of q to δ is essentially cubic, a fact that allows the dynamic response of the strip to an instantaneously applied load to be characterized.

The deflections of the membrane strip satisfy the following equation³ developed for a stretched, laterally loaded flexible string. S is the horizontal (x direction) component of the stress resultant in the membrane:

$$\frac{d^2w}{dx^2} = \frac{q}{S} \quad (1)$$

Since S is independent of x , the deflection curve of the strip is a parabola:

$$w = \frac{\delta}{l^2} x^2 - \delta \quad (2)$$



Let x' be a coordinate along the arc of the deflected surface with origin at $(0, -\delta)$ of the xw axes.

$N_{x'}$ = Stress Resultant in Membrane x' direction.

$\epsilon_{x'}$ = Strain in Membrane x' direction.

$u_{x'}$ = Displacement along Membrane x' direction.

From the diagram:

$$N_{x'} = \sqrt{S^2 + S^2 \left(\frac{dw}{dx} \right)^2} \quad (3)$$

Substituting for dw/dx , we obtain the relation between the stress resultant and the deformed geometry:

$$N_{x'} = S \sqrt{1 + \frac{4\delta^2 x^2}{l^4}} \quad (4)$$

The relation between the strain and the deformed geometry is:

$$\epsilon_{x'} = \epsilon_{x'}^0 \sqrt{1 + \frac{4\delta^2 x^2}{l^4}} \quad \epsilon_{x'}^0 = \frac{S}{A_{11}} = \epsilon_{x'}|_{x=0} \quad (5)$$

Where A_{11} is extensional stiffness for isotropic material subject to the condition $\epsilon_y = 0$.

The displacement along x' at $x=0$ is 0 and the displacement along x' at $x=l$ is the arclength of the parabola minus l .

$$u_{x'}|_{x=0} = 0 \quad u_{x'}|_{x=l} = L_0^1 - l \quad \text{where } L_0^1 = \int_0^1 \sqrt{1 + \left(\frac{2\delta x}{l^2}\right)^2} dx \quad (6)$$

The strain along x' can be integrated to get displacement along x'

$$u_{x'}|_{x=l} = \int_0^1 \frac{du_{x'}}{dx'} \frac{dx'}{dx} dx \quad \text{where } \frac{du_{x'}}{dx'} = \epsilon_{x'}, \quad \frac{dx'}{dx} = \sqrt{1 + \frac{4\delta^2 x^2}{l^4}} \quad (7)$$

Performing the integration:

$$u_{x'}|_{x=l} = \int_0^1 \epsilon_{x'}^0 \left(1 + \frac{4\delta^2 x^2}{l^4}\right) dx = \epsilon_{x'}^0 \left(1 + \frac{4}{3} \frac{\delta^2}{l}\right) \quad (8)$$

The strain and the stress resultant S can be found from substitution of (6) into (8)

$$\epsilon_{x'}^0 = \frac{L_0^1 - l}{1 + \frac{4}{3} \frac{\delta^2}{l}} \quad S = \frac{A_{11} (L_0^1 - l)}{1 + \frac{4}{3} \frac{\delta^2}{l}} \quad (9)$$

From transverse equilibrium:

$$S \frac{2\delta}{l} - ql = 0 \quad (10)$$

Combining equations (9) and (10) gives:

$$\frac{2\delta A_{11}(L_0^1 - l)}{l^3 + \frac{4}{3}\delta^2 l} = q \quad (11)$$

Equation (11) is a nonlinear algebraic equation for the deflection of the membrane under a given load q . Numerical results can be obtained by iterative solution of (11) with L_0^1 computed by numerical integration.

A closed form integral exists to express L_0^1 :

$$L_0^1 = \frac{2\delta\sqrt{l^4 + 4\delta^2 l^2} + \sinh^{-1}(2\frac{\delta}{l})l^3}{4\delta l} \quad (12)$$

Substituting (12) into (11) and expanding in a Taylor series about $\delta=0$ gives:

$$q = \frac{4 A_{11} \delta^3}{3 l^4} - \frac{116 A_{11} \delta^5}{45 l^6} + \frac{4328 A_{11} \delta^7}{945 l^8} - \dots \quad (13)$$

Nondimensionalizing (13) by defining a nondimensional pressure $\phi = ql/A_{11}$ and a nondimensional deflection $\eta = \delta/l$ gives:

$$\phi = \frac{4}{3} \eta^3 - \frac{116}{45} \eta^5 + \frac{4328}{945} \eta^7 - \dots \quad (14)$$

The expansion is valid for small η since the expansion is about $\eta=0$. The leading term in the expansion is cubic and each subsequent term is higher by two orders. For small η the leading cubic term dominates and the relationship of pressure to deflection is essentially cubic. The difference between the cubic term and (11) is of $o(\eta^5)$.